Symmetry in Nature: Snowflakes and Viruses

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Snowflakes

The shape of snowflakes was first discussed as early as BC 135 in Han Ying's book "Disconnection," in which he contrasts the hexagonal symmetry of the snowflake to the pentagonal symmetry he observed in flowers. In 1611, Johannes Kepler explained his theories on why snowflakes are hexagonal.

The geometry of snowflakes

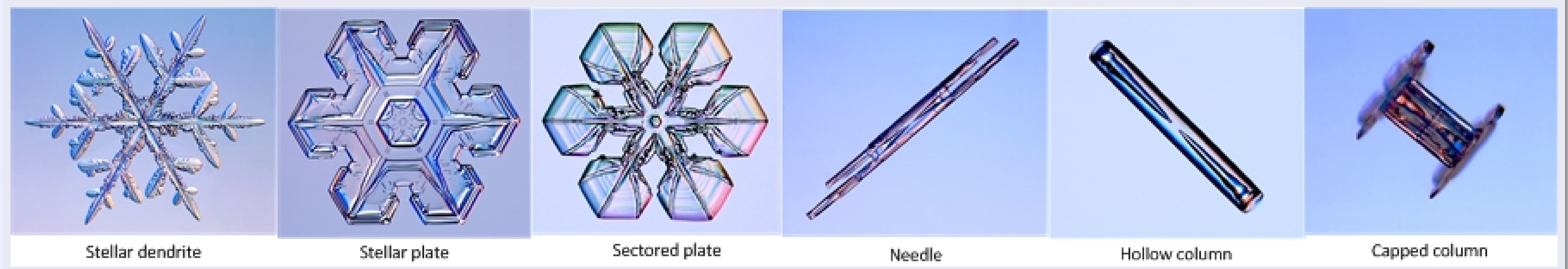
Snowflakes exhibit a rich combination of characteristic *symmetry* and *complexity*:

- Three fold or six-fold symmetry due to geometry of molecular bonds;
- Complexity due to random motion experienced by individual snow crystals through atmosphere.

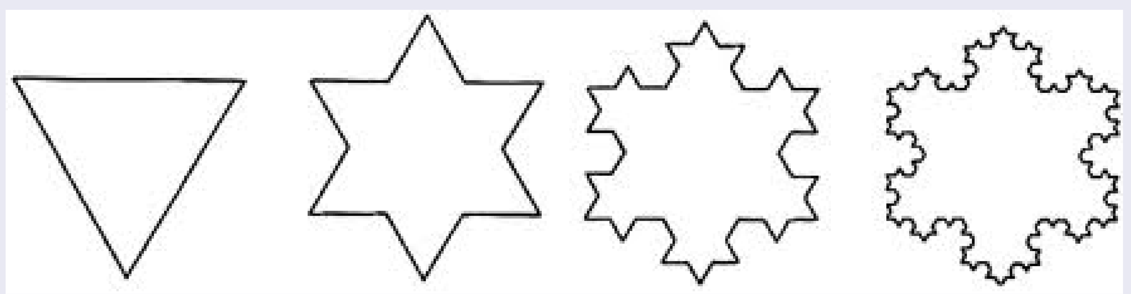
No two Snowflakes are identical in appearance, yet being symmetrical

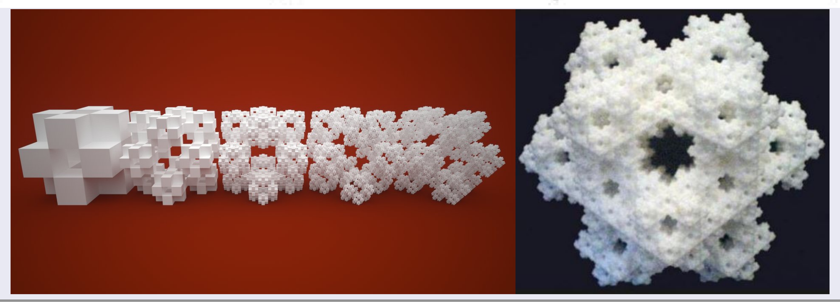
Classification: plates, dendrites, needles, columns, etc.

Snowflake photos by K. Libbrecht [2]



Koch snowflake: self-similarity and fractional dimension

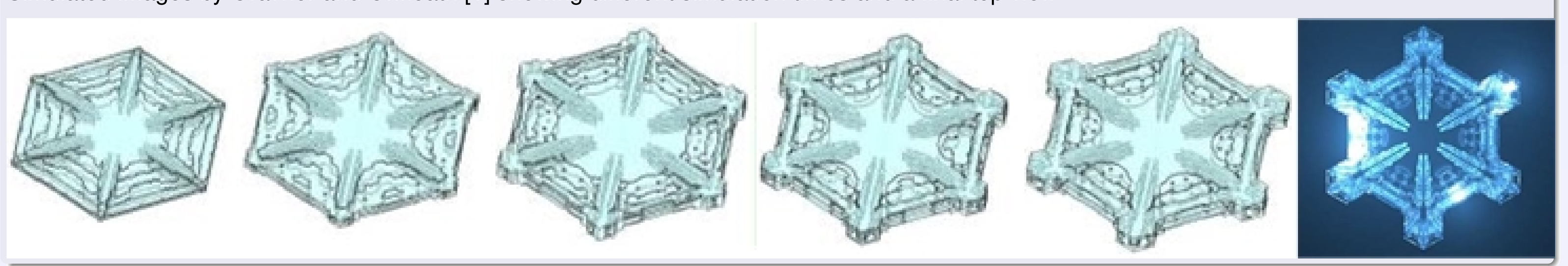




Snowflakes: Computer Modeling

- Through Computer Modeling one can capture essential features with relatively simple mathematical models (diffusion, freezing, attachment, melting, noise) and simulate the models to produce snow crystal images
- Composition of computer-generated images with actual ones helps correlate methematical models and their parameters with physical conditions so as to further our understanding of the physics of snow crystals.

Simulated images by Gravner and Griffeath [1] showing different simulation times and a final top view



Comparison

	Snowflake	Virus
crystal type	crystal	quasicrystal
symmetries	3,6,12 fold symmetry	2,3,5 fold symmetry
mutations	temperature and supersaturation	RNA or DNA
applications	self-similar structure in plants	curing disease

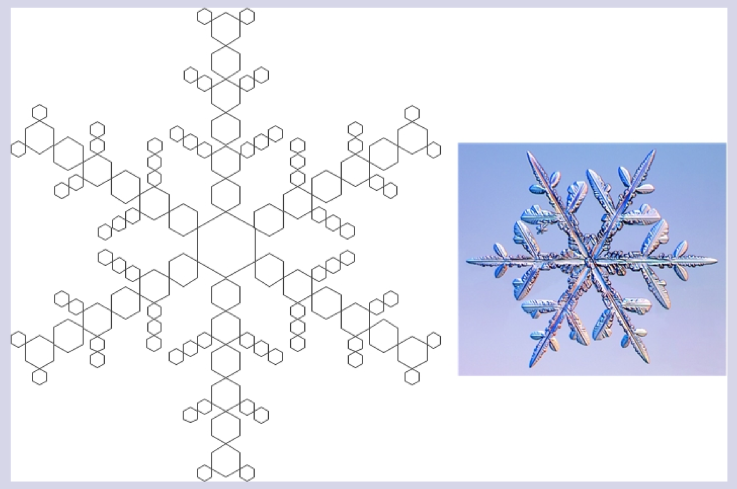
Puzzles

Through Puzzles printed with a 3D Printer, we can see easily basic properties

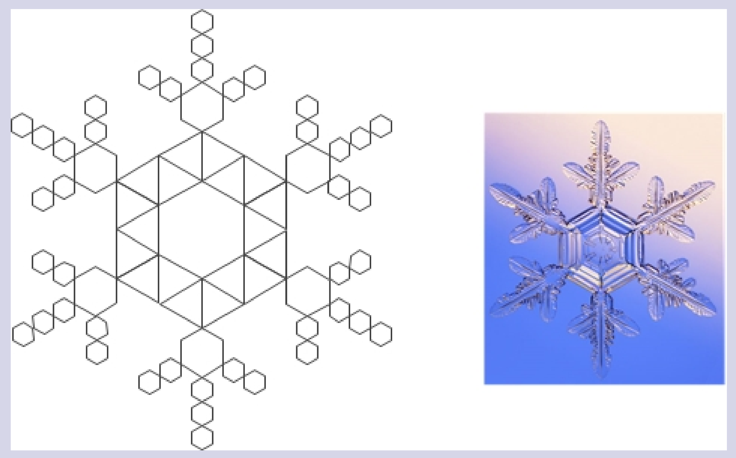
- For snowflakes, we can see that
  - basic structures do not determine the symmetry group of the snowflake;
  - diffusion, freezing and melting influence the construction of the crystals.
- For viruses, we can understand better
  - mutations of viruses;
  - which viruses should exists from a geometry perspective but have yet not been found [4].

Snowflake Puzzles

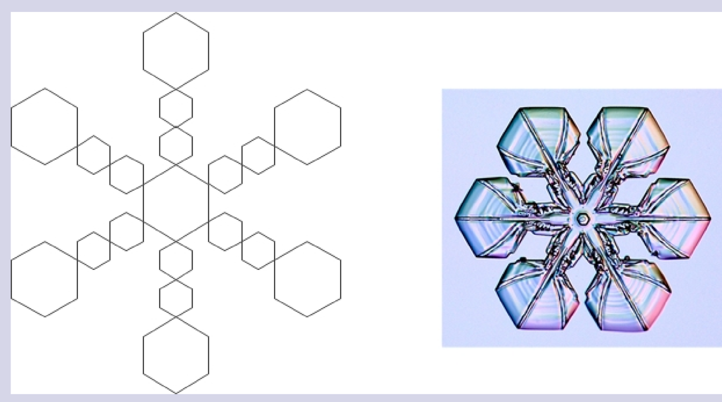
Using fundamental pieces such as hexagons and triangles, we interlock them and produce the different puzzles.



stellar dendrite



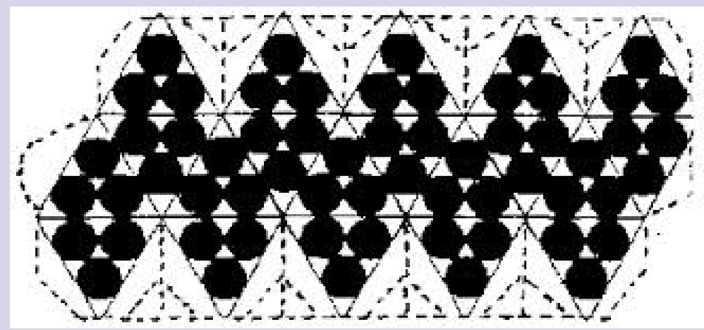
sectored dendrite



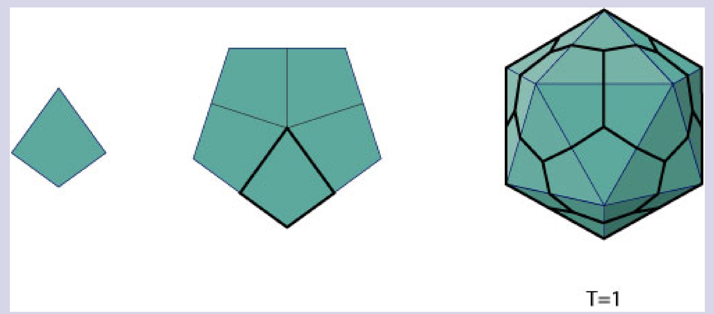
sectored plate

Virus Puzzles

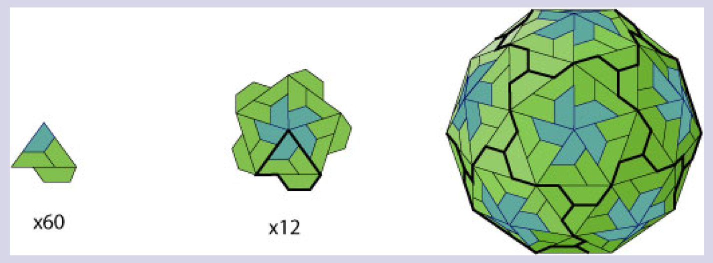
We can use basic pieces to build icosahedral capsid.



Buidling Icosahedral capsid



T=1



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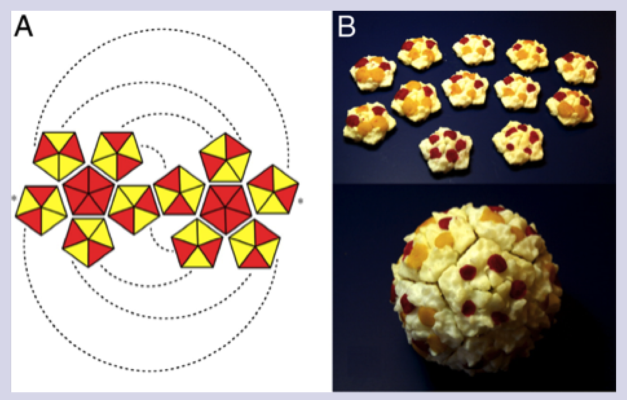
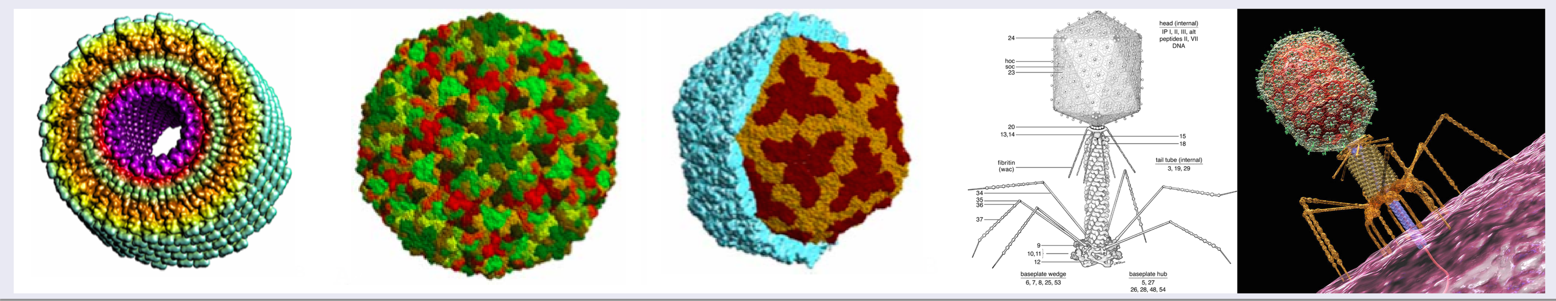


Viruses

In 1956, James Watson and Francis postulated that small viruses are formed from identical proteins that are arranged according to symmetry. This sparked an interest in biologists and other scientists to contact mathematicians to get more insight on possible symmetries the viruses could have and what they could cause to the viruses, in terms of function and replication.

The geometry of Viruses

- Viruses can be classified as helical, icosahedral, or irregular shape by the geometry of their capsid
- ***Helical viruses*** can be described by their diameter and pitch (the distance covered by each complete turn of the helix). The number of *pitch*(P) is the product of the number of protein subunits per helical *turn*(u) and the axial rise per *subunit*(p).
- ***Icosahedral viruses*** have 5-fold, 3-fold, and 2-fold axes
- Viruses have frequency which measure approximate portions of the shell. The frequencies are determined based on line distances from pentagons to pentagons.



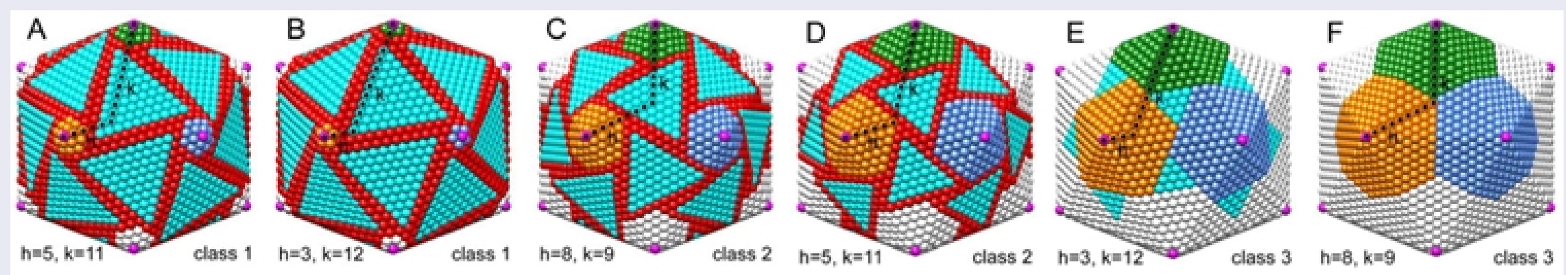
Viruses: Modeling

An icosahedral viral capsid is constructed from N_{cap} capsomers.

Geometric rules: Given e_{PS} = edge length of pentasymmetrons (pentagonal symmetry), e_{TS} = edge length of trisymmetrons (triangular symmetry), T = triangulation number, h, k integers.

- $N_{cap} = 12 + 10(T-1) = 20N_{TS} + 12N_{PS}$
- $N_{PS} = 1 + 5e_{PS}(e_{PS} - 1)/2$
- $N_{TS} = e_{TS}(e_{TS} + 1)/2$
- $T = h^2 + k^2 + hk$
- $e_{TS}(e_{TS} + 1)/2 = (T-1)/2 - 3e_{PS}(e_{PS} - 1)/2$

Leading to three classes of construction rules



Main Bibliography

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